Resolution, depth of field, and perspective.

Stephen Picken, Den Haag, Dec 2007 – Jun 2008, version 21-6-2008

This overview is intended to shed some light on imaging by optical camera systems. It will only deal with "ideal" optics which is reasonable for Leica lenses. Owners of other camera types should proceed at their own risk ☺

I. Back to the basics

First recall the principle of the positive lens. Figure 1 shows the imaging of object O at distance S_O onto an image I at distance S_I by an ideal positive lens of focal length f.

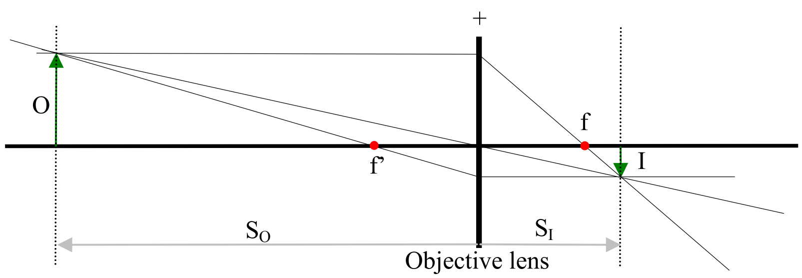


Fig.1: Geometric optics of an ideal positive objective lens.

Depending on the object distance the image plane will be further or closer to the focal plane of the lens. A camera lens is also called an objective or objective lens. In normal photography the object distance is much larger than the focal length and the image will be (very) close to the focal plane of the lens. For many imaging conditions the simple lens formula is sufficient to calculate how this works:

$$\frac{1}{f} = \frac{1}{S_O} + \frac{1}{S_I} \qquad (1)$$

In the case of a camera objective the real world objects are much further away than the focal length f and the image is compressed onto a relatively small digital sensor or photographic film. Note that in microscopy the opposite of this principle is used. In microscopy the object is very close to the focal plane of the microscope objective lens and the image is enlarged onto the image plane, some distance away. In either case the magnification is given by $M=S_I/S_O$.

Example: Using equation 1 for a 50mm lens (i.e. f= 0.05 m) the difference in focussing at S_O = 1 m and 1.05 m corresponds to only 0.132 mm displacement of the objective (the image plane is at respectively 52.500 and 52.632 mm). This explains why a rangefinder camera is an expensive piece of machinery you need high accuracy mechanical coupling of the lens focussing ring to the rangefinder optics. In SLR (single lens reflex) systems this specific problem does not arise as you use the objective itself to focus.

II. What the camera lens markings tell you

Typically a camera objective will be marked with information on the front (fig 2) on:

1. the focal length, typical values are 16, 21, 24, 28, 35, 50, 75, 90 and 135 mm (on old lenses also cm are used: 5 cm is the same as 50 mm obviously),
2. the maximum aperture diaphragm f/value, typically you might see f/1.4, f/2, f/2.8 even f/1, the notation is not fully standardised (f/x is the same as 1:x, f-x, and Fx),
3. sometimes the filter screw diameter, for instance 48 mm is often marked as E 48.

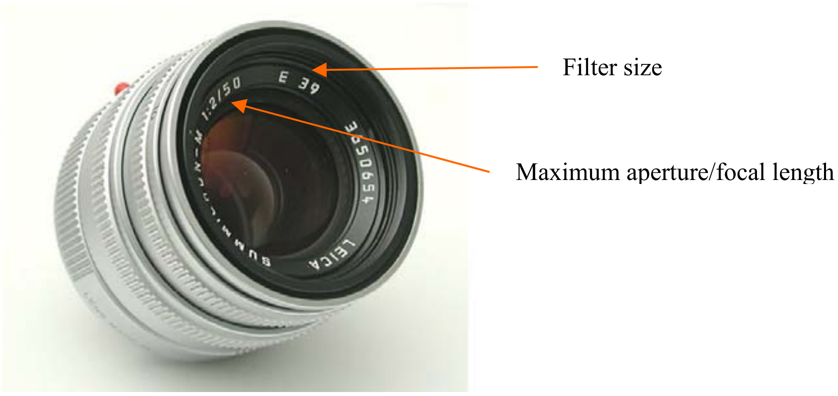


Fig.2: Summicron-M 1:2/50 E 39 lens (50 mm, f/2 maximum aperture, 39 mm filter).

Similarly the lens barrel (fig 3) provides useful information:

4. aperture diaphragm setting = f/value,
5. focussing ring,
6. depth of field scale – this shows the range of distances that are in focus at a given aperture diaphragm f/value, depth of field will be discussed in more detail below,
7. sometimes the focal length, stepwise for Leitz tri-elmar lenses & continuous for zoom.

Fig.3: Leica Summicron-M 1:2/50. This shows the aperture diaphragm, focussing ring and depth of field scale.

What does the maximum diaphragm f/value mean?

The maximum aperture which will be denoted as A_{max} can be derived from the f/value. A_{max} is the ratio of the focal length and the maximum diameter of the lens opening, $A_{max} = f/D$. If the objective is 25 mm in diameter and has a 50 mm focal length it is a f/2 lens, i.e. A_{max} =50/25 = 2. Using this definition we find that a 90 mm f/1 lens would require a primary lens of 90 mm diameter which is impractical - being large, heavy, and expensive to make.

The aperture diaphragm

The aperture can be reduced by closing the aperture diaphragm usually down to A=16 (at f/16) or possibly A=22 (f/22). The usual series for the A values is 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22 etc. each of these so-called stops correspond to a ratio of about √2 (roughly 1.4) in diameter or a factor 2 in light catching surface per stop. Leica lenses often allow ½ stop increments. Some Zeiss lenses have 1/3 stop increments.

The setting of the aperture diaphragm has two consequences i) it governs the amount of light that falls on the film or the sensor; ii) it controls the depth of field. The first effect should be obvious. The further open the diaphragm the more light can enter. In fact each f-stop corresponds to a factor 2 in shutter speed. So correct illumination at f/5.6 and 1/125 s is the same as f/8 and 1/60 s (or f/11 and 1/30 s). In this example to keep the same exposure value the amount of light is reduced by a factor 2 and the shutter time is increase by the same factor.

Again shutter speeds follow a standard series 4, 2, 1, 1/2, 1/4, 1/8, 1/16, 1/30, 1/60, 1/125, 1/250, 1/500, 1/1000 s, each roughly a factor 2 apart (in some cases also intermediate values can be set at ½ stop increments).

The depth of field is a more complicated subject that we will discuss below in more detail. But to set the scene: from the above you might think that f/5.6 at 1/125 s and f/8 and 1/60 s are "the same" but this is not the case! The difference in lens aperture also influences the depth of field of the image. Before we get lost in the gory details it is worth summarising that a ***large f/value like f/1 or f/1.4*** means a ***small depth of field*** and a high light sensitivity while a ***small f/value like f/8 or f/16*** means that ***nearly everything is in focus.***

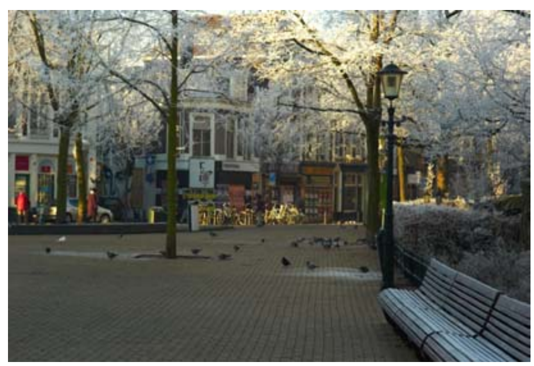
A small aperture gives a large depth of field and vice versa.

This is a very important factor to take into account in photography as it fundamentally changes the appearance of two otherwise identical scenes. This is shown in figure 4a and 4b on a rather uninspiring object. How this effect can be used in a composition is shown in figures 4c and 4d.

4a

4b

4c



4d

Fig. 4a, top-left, 28mm @ f/2 low depth of field; 4b, top-right, 28mm @ f/16 high depth of field. In both cases the string on the balustrade was in focus; 4c, bottom-left, using a small depth of field to isolate the object, 90mm Tele-Elmarit at f/2.8; 4d, bottom-right, using a large depth of field to accentuate the perspective 28mm Summicron ASPH at f/8.

III. Making the depth of field more quantitative

In the preceding we have seen that there are relatively straightforward methods to determine the perfect focus and the amount of light that enters a lens. The depth of field is a slightly more complicated matter. To do this we need to introduce the so-called Numerical Aperture.

Concept 1: the Numerical Aperture N_A

Apart from the aperture f/value there is another standard measure to determine the maximum aperture of an objective which is called the Numerical Aperture. The N_A value is defined as:

$$N_A = n.\sin(\theta_{max}) \tag{2}$$

where θ_{max} is opening angle of the light cone that forms the image and n is the index of refraction of the medium. In nearly all cases occurring in photography n=1 the objective lens is surrounded by air, not by water or oil. Figure 5 shows the geometry to figure out the N_A value.

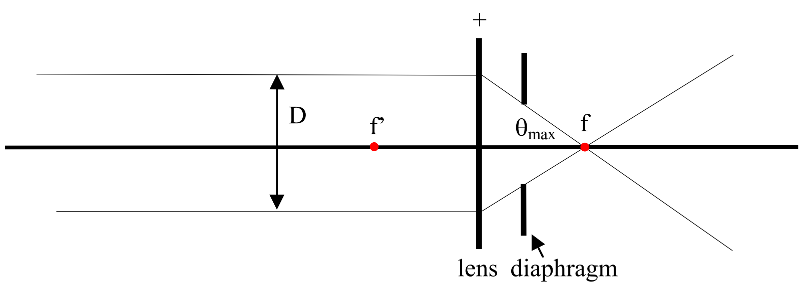


Fig.5: The light cone that defines the Numerical Aperture.

Concept 2: relation of the numerical aperture N_A and the f/value

The value of the numerical aperture N_A can be determined from the f/value. It holds from figure 5 that

$$\tan(\theta_{\max}) = \frac{D}{2f} = \frac{1}{2A}$$

so that N_A is given by

$$N_A = n.\sin(\theta_{\max}) = n.\sin\left(\tan^{-1}\left(\frac{D}{2f}\right)\right) = n.\sin\left(\tan^{-1}\left(\frac{1}{2A}\right)\right) \tag{3}$$

In practical cases for f/value smaller than f/1.4, the relation 3 can be approximated as

$$N_A \approx \frac{D}{2f} = \frac{1}{2A} \tag{4}$$

again using n=1 (in air) and that sin(x) ≈ tan(x) ≈ x for small x values.

By now you might wonder "why are these things important anyway???" The main reason is that the Numerical Aperture of a lens is directly related to the accuracy of forming an image. If we know the aperture f/value then we can calculate the Numerical Aperture and using this we can calculate what accuracy of imaging to expect. This calculation will determine what margin of error is permissible in focussing the lens and this margin of error is called the Depth of Field. Or stated differently: "If I set up my 50 mm lens to focus at 1.6 metres using an aperture f/4 what objects are still approximately focussed?"

Calculating the diffraction limited depth of field – DOF_D.

An ideal lens should focus all light from a single point in the object onto a single point in the image plane. But in real life a lens is not able to do this as it does not capture all light emanating from the object point. This necessarily blurs the image point. From optics theory it is found that the theoretical maximum is governed by diffraction (via so-called Fourier optics) and this yields the result that the focal point is slightly blurry both in width and in depth. A lens with numerical aperture N_A will focus a point roughly of width $\lambda/(2N_A)$ and depth λ/N_A^2 (*note: the exact values depend on the definition used*). So the ideal focal point in real life is a bit fuzzy, see figure 6.

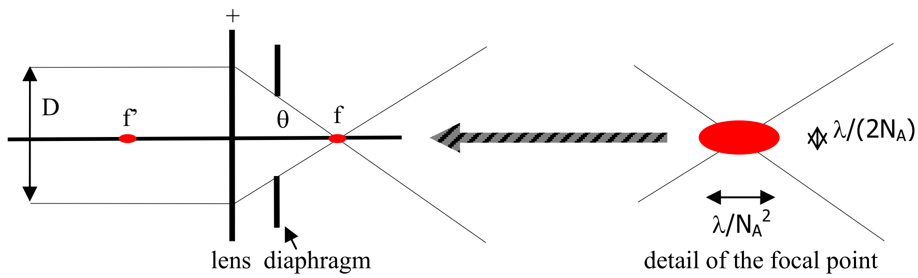


Fig.6: The diffraction limited focal point of a real lens.

Here for simplicity the wavelength λ will be taken to be 500 nm or 0.5 μm which is the average wavelength of visible light. Note that this imaging accuracy, the sharpness of the focal point, is ***independent of the focal length*** of the lens. It only depends on the relative value of the light capturing surface and the focal length of the lens i.e. it only depends on the N_A value. So from a diffraction point of view a 50 mm lens at f/4 and a 200 mm lens at f/4 are just as accurate in their imaging capacity – their focal points are "equally fuzzy". However, this does ***not*** mean that their diffraction limited depth of field is the same. To compare this requires calculating back via equation 1.

If we consider the diffraction limited depth of the focal point $\Delta f_D = \lambda/N_A^2$ we could consider a point at S_i in the range $S_i - \Delta f_D/2$ and $S_i + \Delta f_D/2$ to be "in focus". Then using equation 1 we obtain the diffraction limited depth of field by back calculating the uncertainty of the focal point to the uncertainty of where the object could have been located.

$$\frac{1}{f} = \frac{1}{S_O^{'}} + \frac{1}{S_I + \frac{1}{2}\Delta f_D} \quad \text{and} \quad \frac{1}{f} = \frac{1}{S_O^{''}} + \frac{1}{S_I - \frac{1}{2}\Delta f_D} \qquad (5)$$

The object could be anywhere between $S_O^{'}$ and $S_O^{''}$ giving $DOF_D = S_O^{'} - S_O^{''}$.

It is useful to do a specific calculation as an example. Take a lens with f=35mm, f/5.6, and focus S_O at 3 m. Then using eq.1 we find that the image plane is at S_I = 35.408 mm. The numerical aperture N_A is about 0.09 via eq.4 so that the image depth via λ/N_A^2 becomes about 63 micron. Using eq.5 we then get the result that every object in the range 2.79 and 3.25 m is (equally) in focus, DOF_D = 3.25-2.79 = 46 cm. If we do the same calculation for f=90mm, f/5.6 and S_O=3m we get DOF_D = 3.03 – 2.97 m = 6 cm.

The diffraction limited depth of field is telling us how well a lens could do in principle if there are no other factors to consider. If we compare the results from our calculation above for a 35 mm lens to the lens datasheets from Leica they list a Depth of Field between 2.14 and 5.27 m. Something is still clearly missing from our calculation. Nevertheless, being able to calculate the diffraction limited depth of field is useful. It becomes an important factor at aperture values of f/8 or smaller. Note that in such cases, from a photography point of view, the main concern will be the loss of image sharpness and not the exact value of the depth of field. The DOF is going to be large at f/8 or less anyway.

Calculating the circle of confusion limited depth of field – DOF_C

As was just noted in the previous paragraph, if we compare the diffraction limited DOF_D to the listed performance of an objective we see a large difference with what diffraction theory tells us to expect. The reason for this is that in the majority of cases the performance of the lens is substantially better that that of the detector (CCD chip) or photographic film.

The diffraction limited size of the focal point of a decent lens is the micrometer range whereas a pixel of a CCD chip will typically be much larger – about 50 micrometers or so. Analogously, a perfectly sharp beam of light will spread laterally in a film emulsion limiting what level of detail can be distinguished. This is called the Circle of Confusion criterion (COC) and is the main limiting factor in photography. Even if the lens performs better than the Circle of Confusion we have no obvious way of knowing whether this is the case or not. This information is lost in the imperfect detection by the CCD or the film.

The industry standard COC for a 35mm photographic film camera is 30 μm. For a Leica M8 digital rangefinder sensor a COC of 23 μm seems to be the currently accepted value, this takes the sensor crop factor of (0.75x30=22.5) and the higher print enlargement into consideration. *(Note: the difference in sensor size compared to 35mm film is called the crop-factor usually based on the ratio of the sensor-diagonal/film-diagonal.)* It is important to stress that these COC values are just that: values – something that people have agreed on during the historical development of the art & science of photography.

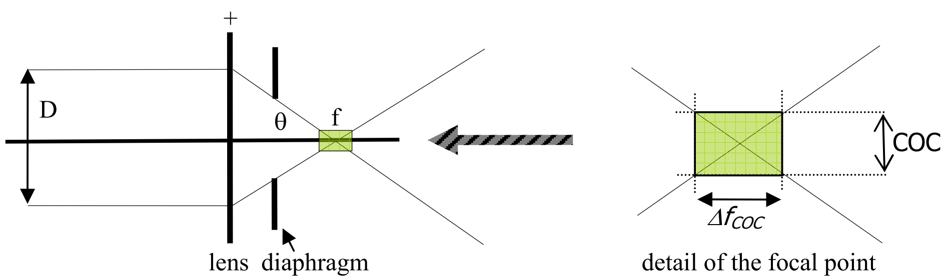


Fig. 7. This shows how the numerical aperture and the Circle of Confusion govern the uncertainty in the location of the focal point

From figure 7 it is clear that a given width of the COC and a given NA also define what is still "in focus" and can derive the Circle of Confusion focal depth:

$$N_A = \frac{½COC}{½\Delta f_{COC}} = \frac{COC}{\Delta f_{COC}} \text{ or rearranging } \Delta f_{COC} = \frac{COC}{N_A} \qquad (6)$$

Then with the Δf_{COC} value we can again use equation 1 to determine the Depth of Field, in this case the DOF_C:

$$\frac{1}{f} = \frac{1}{S_O^*} + \frac{1}{S_I + ½\Delta f_{COC}} \text{ and } \frac{1}{f} = \frac{1}{S_O^+} + \frac{1}{S_I - ½\Delta f_{COC}} \qquad (7)$$

and thus

$$DOF_C = S_O^* - S_O^+ \qquad (8)$$

Again an example: Take a lens with f=35mm, f/5.6, and focus S_O at 3 m. Assume the COC = 31 μm. Then using eqs. 6-8 we find the critical object planes at 2.12 and 5.16 m and DOF_C = 304 cm. Similarly for f=90mm, f/5.6, S_O=3m and COC = 31 μm we get 2.83 and 3.19 m for *the front and back object planes and DOF_C = 36 cm. Comparing this to the tabulated lens data yields 2.14 – 5.27 m and 2.817 – 3.209 m which is looking much better.*

Calculating overall depth of field – DOF

The remaining discrepancies between what we calculate and what a lens table shows can largely be removed by using the overall diffraction and Circle of Confusion limited Depth of Field. The overall accuracy of the focus in mathematical terms is a convolution of the two contributions. As they constitute two independent contributions a reasonable estimate for the overall focus accuracy Δf is obtained from:

$$\Delta f = \sqrt{\Delta f_D^2 + \Delta f_{COC}^2} \qquad (9)$$

Based on this we find for the above examples:
f=35mm, f/5.6, COC = 31 μm focus S_O at 3 m
Calculated DOF = 2.11 – 5.23 m
Listed DOF = 2.102 – 5.294 m

f=90mm, f/5.6, COC = 31 μm focus S_O at 3 m
Calculated DOF = 2.83 – 3.20 m
Listed DOF = 2.817 – 3.209 m

The following table shows calculated values for a 50 mm lens compared to some brochure values at a nominal focus of 2 m, again we see that a reasonably good prediction of lens performance can be obtained from the model.

aperture	*IDEAL 50mm lens*	*Brochure specifications*	
A	*DOF back – front*	*Noctilux*	*Summilux ASPH*
1	*1.95 – 2.05*	*1,954 - 2,049*	*na*
1.4	*1.93 – 2.07*	*1,938 - 2,067*	*1,934 - 2,070*
2	*1.91 – 2.10*	*1,912 - 2,097*	*1,910 - 2,099*
2.8	*1.87 – 2.14*	*1,879 - 2,138*	*1,877 - 2,141*
4	*1.83 – 2.21*	*1,831 - 2,204*	*1,829 - 2,208*
5.6	*1.76 – 2.31*	*1,772 - 2,298*	*1,768 - 2,304*
8	*1.67 – 2.49*	*1,689 - 2,455*	*1,685 - 2,466*
11	*1.57 – 2.77*	*1,597 - 2,685*	*1,591 - 2,704*
16	*1.41 – 3.51*	*1,464 - 3,185*	*1,458 - 3,225*

Finally below we give an Excel spreadsheet (click to open) to calculate the DOF. Here the focusing distance is defined as $S_O + S_I$, the object to film distance, which is the official way that this is done in photography. It has the advantage that this is unambiguously defined whereas the object and image distance become more complicated for real lenses.

Focal length	f	50	mm
Aperture	f/#	1	f/value
Circle of confusion	COC	31	μm
theta-max	θ-max	0.463648	radians
Numerical aperture	NA	0.447214	n.a.
wavelength	λ	500	nm

	Diffraction limited focus depth			**Circle of confusion focus depth**			**Combined focus depth**		
	Δf	2500	nm	Δf	69318.11	nm	Δf	69363.17	nm
lens object distance	So+Si	2	m	So+Si	2	m	So+Si	2	m
object distance	So	1.95	m	So	1.95	m	So	1.95	m
image distance	Si	51.282	mm	Si	51.282	mm	Si	51.282	mm
Magnification	M	0.02632		M	0.02632		M	0.02632	
	1/M	38		1/M	38		1/M	38	
image - delta	Si-Δf/2	51.281	mm	Si-Δf/2	51.247	mm	Si-Δf/2	51.247	mm
image + delta	Si+Δf/2	51.283	mm	Si+Δf/2	51.317	mm	Si+Δf/2	51.317	mm
object max	So-max	2.00	m	So-max	2.05	m	So-max	2.05	m
object min	So-min	2.00	m	So-min	1.95	m	So-min	1.95	m
	Diffraction limited depth of field			**Circle of confusion depth of field**			**Overall depth of field**		
Depth of field	DL-DOF	0.00	m	CC-DOF	0.11	m	DOF	0.11	m

Combining the previous equations and ignoring the effect of diffraction provides a reasonably good estimate for the DOF, which may be more convenient for calculation than a spreadsheet (watch out for the units, everything must me in m or mm but not "mixed"):

$$O^{','} = O\frac{1}{1 \pm \dfrac{COC \cdot O}{2f^2 N_A}} + f = \frac{2f^2 N_A \cdot O}{2f^2 N_A \pm COC \cdot O} + f \qquad (10)$$

Note: Adding f or not to this expression depends on whether you define the focus distance as S_O or as $S_O + S_I$ with respect to the lens position or the film/detector plane.

Example:
From the spreadsheet we get *f=35mm, f/5.6, COC = 31 μm focus* S_O+S_I *at 3 m, calculated DOF = 2.11 – 5.23 m*
From equation 10 we get DOF = 2.12 – 5.16

IV. Some remarks concerning resolution

From the information provides above the resolution of an objective can be easily assessed. If the resolution is limited by diffraction we obtain a width of the focus spot of about $\lambda/(2N_A)$ which is roughly $2A\lambda$ or about 0.5 – 8 μm.
In photography diffraction is very rarely a limiting factor as everything is normally dominated by the circle of confusion at about 20 – 30 μm. This however only holds for high quality objective lenses. In many cases cheaper lenses have a variety of aberrations that will yield visible imperfections in the focus quality most notably the chromatic aberration. This and other concepts, like barrel-distortion, contrast, vignetting - to name but a few, will not be discussed further.

It is worth noting that in microscopy often the imaging is diffraction limited. It is standard practice to image microscopic objects with a resolution better than 1 μm. The highest resolution is reached using oil-immersion objectives with optical immersion-oil between the objective and the sample to enhance the Numerical Aperture.

A CD or DVD player operates in the diffraction limit where the pit-width is about 0.5 μm for a standard CD.

Finally, in computer chip manufacturing with current technology (2007) resolutions of 40nm are possible. This requires an incredible range of techniques: deep UV to reduce the wavelength (at present 193 nm) using CaF_2 aspherical lenses to image this, use immersion technology with water between the lens and the wafer – again enhancing the Numerical Aperture using the fact that n_{water} is about 1.5 for deep UV. Moreover this all operates "on the fly" the wafer and the mask are moving around very quickly in the lithography scanning equipment.

V. Lens Perspective

In the film era comparison of camera performance was relatively easy due to the fact that the *de facto* standard was the 35 mm film format. Larger film formats like 6x6 cm were mainly limited to professional use (like Hasselblad, Rolleiflex) and smaller film formats (110, APS) were never competitors due to the larger amount of grain in the prints. For the serious amateur and indeed for many professional photographers the 35 mm format was the best compromise between quality, weight and indeed price. Suitable for making snapshots during holidays, for serious street-photography, for nature photography – it was all done successfully.

With the digital-era matters have become much more confusing due to the large ranges of CCD chip sizes, number of pixels etc. How should you compare all of this? The main problem here is that there are no fixed standards on how to do such a comparison so a few comments may be useful. However absolute truth does not exist.

In the majority of digital cameras the sensor size is less than that of the 35 mm film. At present (2007) only in a few cameras full-frame 35mm equivalent digital sensors are being used. For such systems the conversion issue does not arise except that the digital sensors usually are sharper and more light-sensitive than photographic film. For such full-frame sensors the same relation between focal length and perspective will hold as in the standard 35mm film format. A 50mm lens has the "same perspective" as the naked eye, shorter focal lengths like 24, 28, and 35 mm are "wide angle" and 70 – 135mm (or further) objectives are long focal length lenses*. One can still argue on what one would like to define as "the same perspective as the naked eye" but again this is a matter of convention. Take a picture with a 50mm lens on 35 mm film, print it on a standard photographic print format and hold it at normal reading distance; the result will look the same as what you originally saw.

A wide angle lens will give a wider frame of view and will tend to increase (stretch) the apparent depth of the scene, similarly a long focal length will give a small magnified view of the world and the image depth is flattened. This can be visualised schematically as follows:

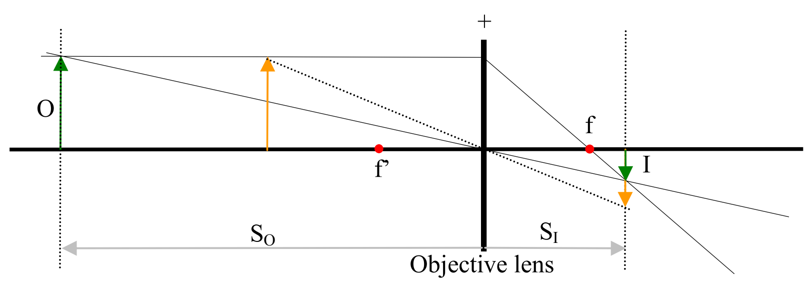


Fig. 8. Medium focal length - the orange arrow is projected about 2x larger than the green one.

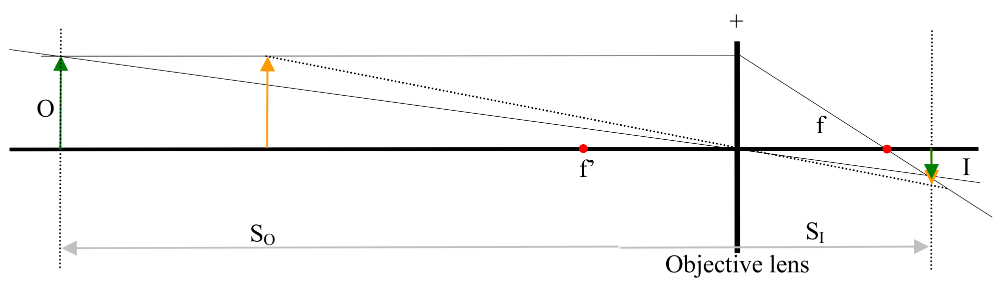


Fig. 9. Long focal length - the orange arrow is projected only slightly larger than the green one.

In both drawings the distance between the objects is the same and the green image has the same size. This reduced sensitivity to image distance variations in a long focal length lens causes the flattening of the perspective. At the opposite extreme a short focal length will grossly exaggerate the perspective.

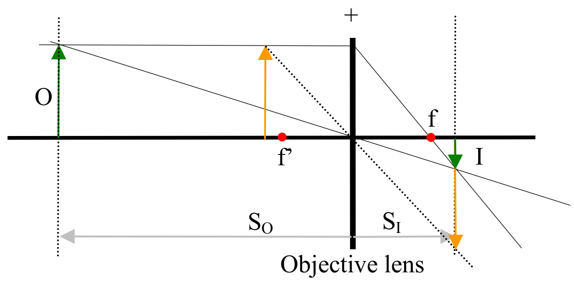


Fig. 10. Short focal length: the orange arrow is projected much larger than the green one.

Note that in the above figures 8-10 the green arrow is in focus, while the orange arrow is outside the focus. Whether its image will appear sharp or fuzzy depends on the Depth of Field discussed earlier.

For some examples see figure 11.

Fig.11. The difference in perspective of a wide-angle and a long focal lens. The two bears are the same size, left 28 mm, right 90 mm lens, on Leica M8 (0.75 crop factor).

* Note: long focal lens lenses are often erroneously referred to as “tele-lenses”. Strictly speaking a telephoto lens is defined as a lens where the lens plane is outside (in front of) the physical lens barrel. So the front element of a 90mm tele-elmarit objective is less than 90mm away from the film/sensor. Obviously a telephoto construction is especially convenient for long focal length objectives as it makes the lens more compact. The inverse of a telephoto lens is called a retrofocus lens. This used for extreme wide angle objectives. See e.g. Wikipedia for more information.

VI. The effect of sensor size

How does the focal length vs. crop-factor work? As mentioned previously in many digital cameras the sensor is smaller than a 35mm negative this difference in size is called the crop-factor usually based on the ratio of the sensor-diagonal/film-diagonal. To compare film photography to digital photography this crop-factor has to be taken into account.

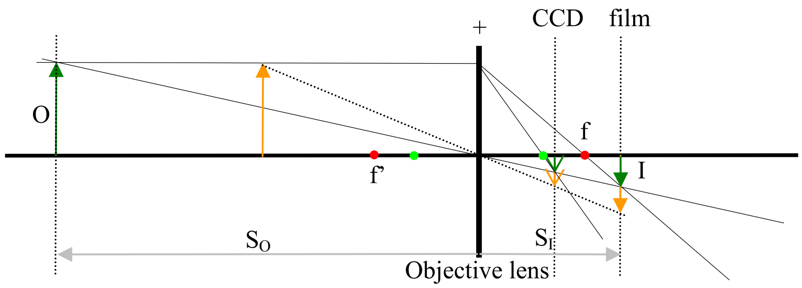


Fig.12. This overlays the imaging by two lenses with a different focal length (red and green focal points). Note that the ratio of the image sizes (yellow and green arrows) remains the same. The longer focal length is assumed to be that of the film objective and the shorter focal length is used to project the same image on a smaller CCD chip.

The schematic drawing shows that if you reduce the image size (smaller sensor) it is still possible to find a lens that images the green and orange arrows in the same ratio i.e. with the same perspective view. Note that to achieve this requires the lens itself to remain in the same position with respect to the objects.

So what do we have in this case? The image is smaller by the crop factor C and the lens must not move so the magnification M = S_I/S_O becomes M' = C M therefore the focal length has to change as prescribed by a reorganised version of eq.1:

$$\frac{1}{f}=\frac{1}{S_O}+\frac{1}{S_I} \text{ or } \frac{S_O}{f}=1+\frac{S_O}{S_I}=1+\frac{1}{M} \quad (11)$$

which gives

$$f=\frac{S_O}{1+\frac{1}{M}} \text{ and } f_C=\frac{S_O}{1+\frac{1}{CM}} \quad (12)$$

If we then assume that the magnification normally is small we can simplify further, finally yielding:

$$f=MS_O \text{ and } f_C=CMS_O \quad (13)$$

This means that if we have a sensor crop of 50% (C=0.5) a lens of C.f or in this specific example half the normal focal length will give the same perspective. So a 50 mm lens on 35mm film is the same as a 25mm lens on a 50% cropped sensor, at least from the perspective point of view.

The ultimate question is will the images look the same???

There are various ways of analysing this which hopefully will clarify the situation.

Analysis 1: constant perspective
Here the DOF calculation comes to rescue. In this hypothetical example let's assume we are focussing on something 2 m away. Then a 50 mm lens at an f/value of f/2 and COC of 31 μm will have a DOF = 1.91 – 2.10. Replacing the 35mm film camera with a 50% cropped digital sensor at f=25mm, a COC of 20 μm and the same aperture gives the depth of field as 1.77 – 2.30. ***So the perspective may be the same but the depth of field will be different.*** *To get something close to the 35 mm depth of field we need larger aperture of about f/1. See also the appendix on calculating the equivalent aperture.*

Analysis 2: cropping the film
The other way to look at the effect of a cropped sensor for a given lens type is that nothing changes except that the image plane is smaller than for a 35mm film. In fact, the resolution is better than with film so if desired you can get better (sharper) results. You are getting closer to what can be achieved by your expensive lens than before. In this interpretation from a perspective and Depth of Field point of view a 50mm lens remains a 50mm lens - with a cropped sensor you only chop the edges of the photo off.

So the answer is "No, they will not look the same." A digital camera and a film camera will only become equivalent if the sensor has the same size as the film.
So which analysis is "true"? From my personal point of view the alternative analysis 2 is just as valid as analysis 1. Indeed, this may explain why with a Leica M8 (with 0.75 crop factor) the 75 mm lens is still popular for portrait photography. Just like was the case for 35 mm film rangefinder cameras. When comparing perspective, resolution and depth of field there is no clear-cut answer of what is better or worse.

VII. Further reading

An alternative text on depth of field can be found here for the "thick lens" but without including diffraction: http://www.vanwalree.com/optics/dofderivation.html

There are various websites on perspective see for instance:
http://www.photozone.de/4Technique/compose/focal.htm
http://www.photozone.de/3Technology/demos/focalCompress.htm

Some articles by Harold Merklinger originally from Shutterbug magazine (thanks to Marco for posting the reference on the l-camera forum):
http://jimdoty.com/Tips/Depth_of_Field/More_DOF/dof_merklinger/dof_merklinger.html

Appendix: Calculating the equivalent aperture

If we have a cropped sensor camera and a certain lens then how should it be compared to a full frame camera and lens? From a perspective point of view we need to compensate the focal length for the crop factor yielding $f = MS_O$ and $f_C = CMS_O$ as discussed previously.

What about the aperture? What aperture setting will then look the same? If we take the ***approximate*** equation 10 then the ratio $COC/f^2.N_A$ has to be the same to get the same depth of field, so it depends on the crop factor and the appropriate value of the Circle of Confusion. Simplifying further the ratio $COC/f^2.N_A$ is roughly equal to $COC.A/f^2$. One approach to obtain the Circle of Confusion for a cropped sensor is tot take $COC_C = COC_{FF}.C$. If we fill this in we find that $COC_C A_C/f_C^2 = COC_{FF}.C.A_C/C^2f^2 = COC_{FF}.A/f^2$ which means that to get the same perspective the aperture A has to become $A_C C$ on the cropped sensor camera. For a Leica M8 this would imply that using an 28/f2 lens is roughly 'equivalent to a (cheaper) 35/2.8 lens on a M7, so you lose about one stop in aperture from the depth of field point of view (not in light capturing efficiency!). This rule of thumb is the approach that is frequently used & cited on the internet (often based on the article "Form Follows Format", Peter Karbe, Head of the Leica Optics Department, LFI 3/2006, pp 40-47).

However, it is worthwhile to analyse the situation more carefully as the equivalent aperture is very sensitive to minor differences is the used COC and moreover it is necessary to do the full calculation to avoid artefacts form the approximations used to derive eq.10. Without going into all the details it is instructive to examine the table below.

	full calc. COC 20		approx. calc. COC 20		
f/value FF	f/value M8	ratio FF/M8	f/value M8	ratio FF/M8	f/value eq.14
1	0.83	1.20	0.83	1.20	0.837
1.4	1.19	1.18	1.19	1.18	1.196
2	1.72	1.16	1.72	1.16	1.726
2.8	2.41	1.16	2.42	1.16	2.429
4	3.45	1.16	3.47	1.15	3.479
5.6	4.81	1.16	4.86	1.15	4.876
8	6.77	1.18	6.93	1.15	6.971

	full calc. COC 23.5		approx. calc. COC 23.25		
f/value FF	f/value M8	ratio FF/M8	f/value M8	ratio FF/M8	f/value eq.14
1	0.67	1.49	0.67	1.49	0.673
1.4	0.99	1.41	0.99	1.41	0.997
2	1.46	1.37	1.46	1.37	1.463
2.8	2.07	1.35	2.07	1.35	2.074
4	2.97	1.34	2.97	1.34	2.982
5.6	4.18	1.34	4.18	1.34	4.187
8	5.96	1.34	5.96	1.34	5.991

Table A1: Calculated apertures to compare a 26.2mm (M8 equivalent focal length) and a 35mm full-frame/film format. Crop factor 0.75 => frame factor 1.33. The Circle of Confusion used was either 31x0.75 = 23.5 or 20 μm.

From the table we can conclude a few things: First that there is little difference in using the approximate eq. 10 or the full calculation especially if we use the crop factor based COC value of 23.5. For the 'standard' value of COC = 20 mm there are subtle differences due to factors that do not quite cancel anymore. Secondly it is clear that the rule of thumb is appropriate for the COC = 23.5 data, for the smaller apertures 2.8 – 8, but that is wrong for the larger apertures f/1 – f/2. Finally it is worth noting that the outcome is substantially different for COC = 20 mm, and is a lot less than 1.33 (the value proposed in 'form follows format').

Using equation 10, it is possible to derive an analytic expression for the effective aperture, which yields:

$$A_c = \tfrac{1}{2}\sqrt{C^4\left(\frac{COC_{35}}{COC_C}\right)^2(1+4A^2)-1} \qquad (14)$$

Here A is the lens aperture, A_C is the effective aperture due to use of a cropped sensor, C is the crop factor, COC_{35} is the full frame Circle of Confusion, and COC_C is the sensor Circle of Confusion. This is also shown in the table and yields the same values for the effective aperture A_C. There are minor deviations at higher aperture values, due to numerical errors and the fact that the full calculation also takes diffraction into account.

So in conclusion - a cropped sensor causes some loss of Depth of Field which can be estimated successfully using equation 14. For the Leica M8 it is as if you are using a 15–20% larger f/value. The exact answer depends strongly on the exact criterion for the circle of confusion of the sensors (or films) being compared. The value to use for these numbers itself is open to discussion. The rule of thumb that the required equivalent aperture is proportional to the crop factor is an approximation especially for larger apertures. This rule of thumb should not be used for detailed analysis of lens performance but is appropriate as a rough estimate.